A Combined Backstepping and Small-Gain Approach to Robust Adaptive Fuzzy Control for Strict-Feedback Nonlinear Systems

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Abstract—In this paper, a robust adaptive tracking control problem is discussed for a general class of strict-feedback uncertain nonlinear systems. The systems may possess a wide class of uncertainties referred to as unstructured uncertainties, which are not linearly parameterized and do not have any prior knowledge of the bounding functions. The Takagi–Sugeno type fuzzy logic systems are used to approximate the uncertainties. A unified and systematic procedure is employed to derive two kinds of novel robust adaptive tracking controllers by use of the input-to-state stability (ISS) and by combining the backstepping technique and generalized small gain approach. One is the robust adaptive fuzzy tracking controller (RAFTC) for the system without input gain uncertainty. The other is the robust adaptive fuzzy sliding tracking controller (RAFSTC) for the system with input gain uncertainty. Both algorithms have two advantages, those are, semi-global uniform ultimate boundedness of adaptive control uncertainty. Both algorithms have two advantages, those are, semi-global uniform ultimate boundedness of adaptive control system in the presence of unstructured uncertainties and the adaptive mechanism with minimal learning parameterizations. Four application examples, including a pendulum system with motor, a one-link robot, a ship roll stabilization with actuator and a single-link manipulator with flexible joint, are used to demonstrate the effectiveness and performance of proposed schemes.

Index Terms—Backstepping technique, fuzzy control, robust adaptive control, robust adaptive sliding control, small gain approach, uncertain nonlinear systems.

I. INTRODUCTION

In practical control engineering, plants are always nonlinear and contain uncertain elements. During the past few years, there have been a lot of researches in the control of highly uncertain nonlinear systems and significant progress has been made in adaptive control schemes for uncertain nonlinear systems via feedback linearization, which has evolved as a powerful methodology. The fundamental concept of feedback linearization [1] is to transform a nonlinear system into a linear one. Therefore, the linear control techniques can be used to acquire the desired performance.

The uncertain nonlinear systems may be subject to the following two types of uncertainties: structured uncertainties, which are referred to as parametric uncertainties, and unstructured uncertainties, which are from modeling errors and external disturbances. In the early stage of the research, the adaptive control methods based on the feedback linearization approach were applied only to a relatively simple class of nonlinear systems. One of the key requirements is that the unknown nonlinearities appear on the same equation as the control input in state space model. Such restriction on the appearance of the uncertain nonlinear functions is usually referred to as matching conditions. We can classify the restrictions on the uncertain nonlinear systems into four cases, namely, matching conditions with structured uncertainties, matching conditions with unstructured uncertainties, nonmatching conditions with structured uncertainties and nonmatching conditions with unstructured uncertainties.

In order to account for the matching conditions with structured uncertainties, the adaptive control method via feedback linearization can be used, which has undergone rapid developments in the past decade, e.g., [2]–[5]. As for the matching conditions with unstructured uncertainties, deterministic robust control method, e.g., [6]–[9], can be used, if there is prior knowledge of the bound on the unstructured uncertainties. Unfortunately, in industrial control environment, there are some plants with the unstructured uncertainties where none of prior knowledge of the bound is available, then the adaptive control method and the deterministic robust control method can not be used to design controller for those systems. A solution to those problems was presented that the neural networks (NN) were used to approximate the continuous unstructured uncertain functions in the systems and Lyapunov’s stability theory was applied in designing adaptive NN controller. Several stable adaptive NN control approaches are developed by [10]–[16], which guarantee uniform ultimate boundedness in the presence of both unstructured uncertainties and unknown nonlinearities. From a mathematical control perspective, fuzzy logic systems can be also used in a similar setting with neural networks. The fuzzy logic systems are used to uniformly approximate the unstructured uncertain functions in the designed system by use of the universal approximation properties of the certain classes of fuzzy systems, which were proposed in [17], [18], a Lyapunov-based learning law was used, and several stable adaptive fuzzy controllers that ensure the stability of the overall system were developed in [19]–[24]. Recently, an adaptive fuzzy-based controller combined with VSS and $H_\infty$ control technique has been studied by Chen et al. [25], Chang [26].

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However, some physical systems may be subject to some uncertainties which do not satisfy restrictions with the matching conditions. In an attempt to overcome the restrictions with nonmatching conditions, a novel recursive design procedure, adaptive backstepping technique, has been presented for a class of nonlinear systems translatable to a parametric strict-feedback canonical ones, e.g., [27]–[29]. However, all those adaptive controllers deal with the case of parametric uncertainties only. The system’s nonlinearities are assumed to be known while parameters are unknown and linear with respect to those known nonlinear functions. Unfortunately, in industrial control environment, there are some plants which are not only characterized by the nonmatching conditions, but also by the unstructured uncertainties which cannot be modeled or repeatable. Neural networks and fuzzy logic systems have been found to be particularly useful for those plants. Recently, several stable robust adaptive NN and fuzzy controllers have been studied for strict-feedback canonical systems with unstructured uncertainties, e.g., [30]–[33]. However, the control methods described above have a substantial drawback, that is, a lot of parameters are needed to be tuned in the online learning laws when there are many state variables in the designed systems, particularly by use of fuzzy logic systems adopted by [33], when many rule bases are used in the fuzzy systems which are used to approximate the uncertain nonlinear functions, so that the learning time tends to become unacceptable large for systems of higher order and time-consuming process is unavoidable when the controllers above are implemented.

In this paper, we will present a novel approach to solve aforementioned problem. A new systematic procedure is developed for the synthesis of stable robust adaptive fuzzy controller for strict-feedback nonlinear systems with unstructured uncertainties. Takagi–Sugeno type fuzzy logic systems [34] are used to approximate the unknown unstructured uncertain functions. Two kinds of robust adaptive fuzzy controllers are proposed by use of input-to-state stability (ISS) theory [35] and by combining backstepping technique with generalized small gain approach [36] and robust control strategy. One is the robust adaptive fuzzy sliding tracking controller (RAFTC) for the system without input gain uncertainty. The other is the robust adaptive fuzzy sliding tracking controller (RAFSTC) for the system with input gain uncertainty. The controllers proposed in this paper guarantee semi-global uniform ultimate boundedness in the presence of unstructured uncertainties. The outstanding features of controllers are that 1) they all have the adaptive mechanism with minimal learning parameterizations, no matter how many states in the designed systems are investigated and how many rules in the fuzzy logic systems are used, only 2\eta parameters needed to be adapted online, where \eta is the dimension of the state in the designed systems, such that the burdensome computation of the algorithm can be lightened and it is convenient to realize the algorithm in engineering, and 2) the possible controller singularity problem can be avoided in some of the existing adaptive control schemes with feedback linearization techniques.

This paper is organized as follows. In Section II, we will present some notations and review some necessary definitions of input-state stability (ISS), small gain theorem and preliminary results. Section III contains a description of strict-feedback uncertain nonlinear systems and proposes a motivating problem. In Section IV, a systematic procedure for the synthesis of robust adaptive fuzzy tracking controller (RAFTC) and robust adaptive fuzzy sliding tracking controller (RAFSTC) is developed. In Section V, four applications are used to demonstrate the effectiveness of schemes. The final section contains conclusion.

\section*{Notation}
Throughout this paper, let \( \| \cdot \| \) be any suitable norm. The vector norm of \( x \in \mathbb{R}^n \) is Euclidean, i.e., \( \| x \| = x^T x \) and the matrix norm of \( A \in \mathbb{R}^{n \times m} \) is defined by \( \| A \| = \max(\| A^T \|) \), where \( \lambda_{\max,min}(\cdot) \) denotes the operation of taking the maximum (minimum) eigenvalue. The norm over the space defined by stacking the matrix columns into a vector, so that it is compatible with the vector norm, i.e., \( \| A x \| \leq \| A \| \| x \| \).

For any piecewise continuous function \( u : \mathbb{R}_+ \to \mathbb{R}^m, \| u \|_{\infty} = \sup\{\| u(t) \|, t \geq 0 \} \), it stands for \( \mathcal{L}_{\infty} \) supremum norm, and for any pair of times \( 0 \leq t_1 \leq t_2 \), the truncation \( u_{[t_1, t_2]} \) is a function defined on \( \mathbb{R}_+ \) which is equal to \( u(t) \) on \( [t_1, t_2] \) and is zero outside the interval. In particular, \( u_{[0, t]} \) is the usual truncated function \( u_t \).

\section{Preliminaries}
\subsection{ISS and Small Gain Theorem}
The concepts of ISS and ISS-Lyapunov function proposed by [35], [38], [39] have recently been used in various control problems such as nonlinear stabilization, robust control and observer designs (see, e.g., [40]–[45]). In order to ease the discussion of the design of RAFTC and RAFSTC schemes, two definitions with respect to input-to-state stability are reviewed in the following. First, we recall the class \( K, K_{\infty} \) and KL functions which are standard in the stability literature, see Khalil [46].

A class \( \mathcal{K} \)-function \( \gamma \) is a continuous, strictly increasing function from \( \mathbb{R}_+ \) into \( \mathbb{R}_+ \) and \( \gamma(0) = 0 \). It is of class \( K_{\infty} \) if additionally \( \gamma(s) \to \infty \) as \( s \to \infty \). A function \( \beta : R_+ \times R_+ \to R_+ \) is of class \( KL \) if \( \beta(s, t) \) is of class \( K \) for every \( t \geq 0 \) and \( \beta(s, t) \to 0 \) as \( t \to \infty \).

\textbf{Definition 1}: For the system \( \dot{x} = f(x, u) \), it is said to be input-to-state practically stable (ISpS) if there exist a function \( \gamma \) of class \( K_{\infty} \) gain, and a function \( \beta \) of class \( KL \) such that, for any initial condition \( x(0) \), each measurable essentially bounded control \( u(t) \) defined for all \( t \geq 0 \) and a nonnegative constant \( d \), the associated solutions \( x(t) \) are defined on \( [0, \infty) \) and satisfy

\begin{equation}
\| x(t) \| \leq \beta(\| x(0) \|, t) + \gamma(\| u \|_{\infty}) + d. \tag{1}
\end{equation}

When \( d = 0 \) in (1), the ISpS property reduces to the input-to-state stability (ISS) property introduced in [38].

\textbf{Definition 2}: A \textit{C} \text{\textsuperscript{1}} function \( V \) is said to be an ISpS-Lyapunov function for the system \( \dot{x} = f(x, u) \) if there exist functions \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) of class \( K \) and a constant \( d > 0 \) such that

\begin{equation}
\frac{\partial V(x)}{\partial x} f(x, u) \leq -\alpha_1(\| x \|) + \alpha_4(\| u \|) + d. \tag{2}
\end{equation}

\begin{equation}
\alpha_1(\| x \|) \leq V(x) \leq \alpha_2(\| x \|), \forall x \in \mathbb{R}^n \tag{3}
\end{equation}
When (3) holds with \( d = 0 \), \( V \) is referred to as an ISS-Lyapunov function.

Then it holds that one may pick a nonlinear \( L_\infty \) gain \( \gamma \) in (1) of the form [40]
\[
\gamma(s) = \alpha_1^{-1} \circ \alpha_2 \circ \alpha_3^{-1} \circ \alpha_4(s), \quad \forall s > 0.
\] (4)

The following proposition establishes equivalence between ISPs and the existence of ISPS-Lyapunov function by Sontag and Wang [39] and Praly and Wung [44].

**Proposition 1:** The system \( \dot{x} = f(x, u) \) is ISPs if and only if there exists an ISPS-Lyapunov function.

A trivial refinement of the proof of the generalized small gain theorem given by [36], [45] yields the following variant.

**Theorem 1:** Consider a system in composite feedback form
\[
\Sigma_{\infty}: \left\{ \begin{array}{l}
\dot{x} = f(x, \omega) \\
\dot{\zeta} = H(x) 
\end{array} \right.
\] (5)
\[
\Sigma_{\infty}: \left\{ \begin{array}{l}
\dot{y} = g(y, \zeta) \\
\omega = K(y, \zeta)
\end{array} \right.
\] (6)

of two ISPs systems. In particular, there exist two constants \( d_1 > 0 \), \( d_2 > 0 \), and let \( \beta \) and \( \kappa \) of class \( KL \), and \( \gamma_2 \) and \( \gamma_{\omega} \) of class \( K \) be such that, for each \( \omega \) in the \( L_\infty \) supremum norm, the \( \zeta \) in the \( L_\infty \) supremum norm, all the solutions \( X(x; \omega, t) \) and \( Y(y, \zeta; t) \) are defined on \([0, \infty)\) and satisfy, for almost all \( t \geq 0 \)
\[
\| H(X(x; \omega, t)) \| \leq \beta_{\omega}(\| x \|_\infty) + \gamma \omega(\| \omega \|_\infty) + d_1
\] (7)
\[
\| K(Y(y, \zeta; t)) \| \leq \beta_{\zeta}(\| y \|_\infty) + \gamma_{\zeta}(\| \zeta \|_\infty) + d_2
\] (8)

Under these conditions, if
\[
\gamma_{\omega}(\gamma_{\omega}(s)) < s \quad (\text{resp. } \gamma_{\omega}(\gamma_{\omega}(s)) < s) \quad \forall s > 0
\] (9)
then the solution of the composite systems (5) and (6) is ISPS.

**B. T-S Fuzzy Systems**

From mathematical point of view, fuzzy logic systems are just practical function approximators. In the past few years, various types of fuzzy logic systems (e.g., Mamdani type and Takagi-Sugeno type) have been proved to be universal approximators in that they can uniformly approximate any continuous functions defined on compact domains to any degree of accuracy. In this subsection, we briefly describe the structure of fuzzy logic systems.

Let \( S \) be a compact simply connected set in \( R^n \). With map \( f : S \rightarrow R^n \), define \( C^m(S) \) to be the function space such that \( f \) is continuous. Consider a fuzzy logic system to uniformly approximate a continuous multidimensional function
\[
y = f(x), \quad x = (x_1, x_2, \ldots, x_n)T.
\]
\( f \) is defined on \( [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n] \). It follows that \( x \) is on
\[
\Phi = \Phi_1 \times \Phi_2 \times \cdots \times \Phi_n = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n].
\]

In order to construct a fuzzy system, the interval \([a_i, b_i]\) is divided into \( N_i \) subintervals
\[
a_i = C_0^i < C_1^i < \cdots < C_{N_i-1}^i < C_N^i = b_i, 1 \leq i \leq n.
\]

On each interval \( \phi_i(1 \leq i \leq n) \), \( N_i + 1(N_i > 0) \) continuous input fuzzy sets, denoted by \( \Psi_j^i(0 \leq j \leq N_i) \), are defined to fuzzify \( x_i \). The membership function of \( \Psi_j^i \) is denoted by \( \mu_j^i(x_i) \), which can be represented by triangular, trapezoid, generalized bell or Gaussian type and so on.

Generally, the fuzzy system can be constructed by the following \( K(\bar{K} > 1) \) fuzzy rules
\[
R_k: \text{If } x_1 \text{ is } \Psi_{j_1}^1 \text{ AND } x_2 \text{ is } \Psi_{j_2}^2 \text{ AND } \cdots \text{ AND } x_n \text{ is } \Psi_{j_n}^n \text{ THEN } y_k \text{ is } \Omega_{j_1j_2\ldots j_n}^k, \quad i = 1, 2, \ldots, K
\]

where \( \Omega_{j_1j_2\ldots j_n}^k \) denotes an output fuzzy set. If \( \Omega_{j_1j_2\ldots j_n}^k \) is a singleton fuzzy set, its membership function is 1 only at \( y_j = \gamma \) (an arbitrary unknown constant) and 0 at other position, then that is called type fuzzy system. If \( \Omega_{j_1j_2\ldots j_n}^k \) is a function of \( a_{j_1}x_1 + a_{j_2}x_2 + \cdots + a_{j_n}x_n \), which \( a_{j_i} \), \( i = 1, 2, \ldots, K \), \( j = 1, 2, \ldots, n \) are the unknown constants, then that is called Takagi–Sugeno type fuzzy system, T-S fuzzy system for short. The product fuzzy inference is employed to evaluate the AND’s in the fuzzy rules. After being defuzzified by a typical center average defuzzifier, the output of the fuzzy system in the vector form
\[
\text{Mamdani type } \hat{f}(x, \sigma) = \xi(x)\sigma
\]
\[
\text{T-S type } \hat{f}(x, A_x) = \xi(x)A_x
\]

where \( \xi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_K(x)] \) and \( \xi_i(x) = \sum_{j=1}^{K} \mu_j^i(x)/\sum_{j=1}^{K} \mu_j^i(x) \), \( i = 1, 2, \ldots, K \) called as fuzzy basis function. When the membership function \( \mu_j^i(x) \) in \( \xi_i(x) \) is denoted by some type of membership function. In (10) and (11), we have \( \sigma = [\sigma_1, \ldots, \sigma_K]^T \) and
\[
A_x = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{K1} & a_{K2} & \cdots & a_{Kn}
\end{bmatrix}.
\]

For any continuous function \( f(x) \), there exists some type fuzzy system to approximate it to an arbitrary accuracy. Then we have the following lemma proved by Wang [37].

**Lemma 1:** Suppose that the input universal of discourse \( U \) is a compact set in \( R^n \). Then, for any given real continuous function \( f(x) \) on \( U \) and \( \forall \varepsilon > 0 \), there exists a fuzzy system \( F(x) \) in the form of (10) or (11) such that
\[
\sup_{x \in U} \| f(x) - F(x) \| \leq \varepsilon
\]
where \( F(x) = \hat{f}(x, \sigma) \) in Mamdani type or \( F(x) = \hat{f}(x, A_x) \) in T-S type fuzzy system.

**Remark 1:** For any \( n \)-dimensional continuous function \( f(x) \), if \( N_i + 1 \) input fuzzy sets for each variable \( x_i \) are used, there will be \( K = \prod_{i=1}^{n} (N_i + 1) \) IF-THEN fuzzy rules in the fuzzy system. If we use Mamdani type or T-S Type fuzzy system to approximate the function \( f(x) \), from (10) and (11), we observe that there are a total of \( \prod_{i=1}^{n} (N_i + 1) \) parameters to describe Mamdani type fuzzy system and \( n \cdot \prod_{i=1}^{n} (N_i + 1) \) parameters to describe T-S type fuzzy system.
III. MOTIVATING PROBLEM

Consider an uncertain nonlinear dynamic system in the following form

\[
\begin{aligned}
\dot{x}_i &= x_{i+1} + f_i(x, w) + d_i, & 1 \leq i \leq n-1 \\
\dot{x}_n &= f_n(x) + f_n(x, w) + [g_0(x) + g_n(x, w)]u + d_n \\
y &= x_1 
\end{aligned}
\]  

(13)

where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^m \) is the system state, \( u \in \mathbb{R} \) is the control input, \( y \in \mathbb{R} \) is the output of system and \( w \) is the model uncertainty belonging to a compact set, which includes uncertain parameter vector of the system. \( d_i, \ i = 1, 2, \ldots, n \) are the disturbance uncertainties of the system. Let \( \varpi_i = [x_1, x_2, \ldots, x_i]^T \). \( f_0(x) \) and \( g_0(x) \) are known functions belonging to smooth vector fields in a neighborhood of the origin \( x = 0 \) with \( f_0(0) = 0 \) and \( g_0(x) \neq 0 \). \( f_i(x, w), \ i = 1, 2, \ldots, n \) are the system uncertain functions with \( f_i(0, w) = 0 \) and \( g_n(x, w) \) is the control input uncertain function, all of which are continuous functions depending on the state \( x \).

The following assumption is introduced.

**Assumption 1:** The uncertain control gain function \( g_n(x, w) \) is confined within a certain range such that

\[
0 < b_{\min} \leq g_0(x)^{-1} g_n(x, w) \leq b_{\max} \tag{14}
\]

where \( b_{\min} \) and \( b_{\max} \) are the lower and upper bound parameters, respectively.

The primary goal of this paper is to track a given reference signal \( y_d(t) \) while keeping the states and control bounded. That is, the output tracking error \( z_1 = y(t) - y_d(t) \) should be small. The given reference signal \( y_d(t) \) is assumed to be available together with its \( n \) time derivatives, and that \( y_d^{(n)}(t) \) is piecewise continuous. Moreover, the vector \( \varpi_{d(i+1)} = [y_d(0), y_d(1), \ldots, y_d(i)]^T \) is bounded, i.e., for some \( \kappa > 0 \), \( \| \varpi_{d(i+1)} \| < \kappa, i = 1, 2, \ldots, n \). We can define the tracking error vector \( e(t) = x(t) - \varpi_{d(n)} \).

Generally speaking, the control objective is to find an adaptive fuzzy tracking controller for the system (13) in the following form

\[
\begin{aligned}
\dot{\chi} &= \varpi(\chi, \xi(e), e), \quad \chi \in \mathbb{R}^p \\
u &= \psi(\chi, \xi(e), e) \tag{15} \\
\end{aligned}
\]

(16)

where \( \xi(e) \) is a known fuzzy base function vector. In such a way that all the solutions of the closed-loop system (13), (15), and (16) are globally uniformly ultimately bounded. Furthermore, the tracking error vector \( e \) can be rendered small.

From (15) and (16), we can observe that that is a dynamic feedback controller and \( p \) is the dimension of the dynamic part \( \chi \) of the controller. An important quality of the controller is of course the property that the dimension \( p \) of \( \chi \) should be as small as possible, and in particular not dependent on the dimension of the state. Therefore, the dynamic part of the controller is the adaptive law for estimating the unknown parameters on-line and the dimension \( p \) of \( \chi \) is equal to the number of parameters to be estimated. In the conventional adaptive fuzzy controller, the dimension \( p \) of \( \chi \) is equal to the number of parameters to be used for describing the fuzzy logic system, which is employed to approximate the unknown uncertain functions in the designed systems. Wang et al. [33] presented an adaptive fuzzy controller for the system (13). They used the Mamdani type fuzzy systems to approximate the functions \( f_i(x, w), \ i = 1, 2, \ldots, n \) and \( g_n(x, w) \). Also, they assumed that the approximation error and external disturbance are square integrable, however the square-integrable property of the approximation error and external disturbance is difficult to show for given plant and this calculation may require knowledge of plant dynamics. Therefore, according to Remark 1, we know that there are a total of \( \sum_{i=1}^{n-1} \prod_{j=1}^{n} (N_i + 1) \) parameters needed to be estimated in their adaptive fuzzy control scheme. In order to describe the problem clearly, we give an example here, i.e., let us discuss a system which has the order \( n = 3 \) and use \( N_i + 1 = 5 \) to be continuous input fuzzy sets, as a result there will be \( 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 280 \) parameters needed to be estimated in the adaptive fuzzy controller proposed by Wang et al. [33]. From this small example, we can see that there will be a lot of parameters to be estimated in the online adaptive law when there are many state variables in the designed systems and many rule bases in the fuzzy system are used to approximate the uncertain nonlinear functions. It will result in the learning time tends to become unacceptably large and time-consuming process is unavoidable when the controller is implemented.

In this paper, we will present two kinds of robust adaptive control algorithms, including robust adaptive fuzzy tracking control (RAFTC) and robust adaptive fuzzy sliding tracking control (RAFSFTC), with the structure as (15) and (16) for a class of strict-feedback uncertain nonlinear system (13) by use of input-to-state stability theory and by combining backstepping technique with generalized small gain approach. The outstanding feature of the algorithms proposed in this paper is that the dimension \( p \) of \( \chi \) is \( 2n \), no matter how many states in the designed systems are investigated and how many rules in the fuzzy system are used, i.e., for the small example given above, there will be only six parameters needed to be estimated in the derived algorithms.

IV. DESIGN OF ROBUST ADAPTIVE FUZZY TRACKING CONTROL

A. Robust Adaptive Fuzzy Tracking Control

First, we will focus on the case that control gain uncertain term \( g_n(x, w) = 0 \) in the system (13) and get the following transformation for normal system

\[
u = \tilde{g}_0^{-1}(x)(-f_0(x) + v) \tag{17}
\]

where \( v \) is a new control variable. Equation (13) can be rewritten as

\[
\begin{aligned}
\dot{x}_i &= x_{i+1} + f_i(x, w) + d_i, & 1 \leq i \leq n-1 \\
\dot{x}_n &= f_n(x) + f_n(x, w) + v + d_n \\
y &= x_1. 
\end{aligned}
\]

(18)

The backstepping design procedure contains \( n_1 \) steps. At each step, an intermediate control function \( \tilde{g}_k \) shall be developed using an appropriate Lyapunov function \( V_k \). We give the proceeding of the backstepping design as follows.

**Step 1:** Define the error variable \( z_1 = x_1 - y_d \), then

\[
z_1 = x_2 + f_1(x_1, w) + d_1 - y_d \tag{19}
\]
Since \( f_1(x_1, w) \) is an unknown continuous function, according to Lemma 1, T-S fuzzy system \( f_1(x_1, A_1) \) with input vector \( x_1 \in U_{x_1} \) for some compact set \( U_{x_1} \subset \mathbb{R} \) is proposed here to approximate the uncertain term \( f_1(x_1, w) \) where \( A_1 \) is a matrix containing unknown constants. Then \( f_1(x_1, w) \) can be expressed as

\[
\dot{x}_1 = c_{\theta_1} \xi_1(x_1) A_1 y_1 + c_\epsilon \xi_1(x_1) A_1 y_2 + c_{\theta_1} \xi_1(x_1) A_1 y_3 + \epsilon_1 + d_1 \]  

(20)

where \( \epsilon_1 \) is a parameter denoting approximating accuracy. Let

\[
c_{\theta_1} = \| A_1 \|, \quad c_\epsilon = \| A_1 \|, \quad \| A_1 \| = 1, \quad \| A_1 \| \leq 1, \quad \| \omega_1 \| = A_1 \| \omega_1 \|
\]

Substituting (20) into (19), we get

\[
\dot{x}_1 = c_{\theta_1} \xi_1(x_1) y_1 - \dot{y}_d + \nu_1
\]

(21)

where \( \nu_1 = \xi_1(x_1) A_1 y_1 + \epsilon_1 + d_1 \) and \( c_{\theta_1} \) is an unknown constant, and there exists a bound for \( \nu_1 \) as follows

\[
\| \nu_1 \| \leq \| \xi_1(x_1) A_1 y_1 + \epsilon_1 + d_1 \| \leq \| \theta_1 \| (x_1) \| \nu_1 \|
\]

(22)

where \( \theta_1 = \max(\| A_1 y_1 \|, \| \epsilon_1 + d_1 \|) \) and \( \psi_1(x_1) = 1 + \| \xi_1 \| \)

Consider the stabilization of the subsystem (21) and the Lyapunov candidate function

\[
V_1(z_1, \lambda_1, \dot{\lambda}_1) = \frac{1}{2} z_1^2 + \frac{1}{2} \gamma_1^2 \lambda_1^2 + \frac{1}{2} \gamma_1^2 \dot{\lambda}_1^2
\]

(23)

where \( \Gamma_{11} \) and \( \Gamma_{21} \) are the positive definite constants. \( \hat{\lambda}_1 = (\theta^{\hat{\lambda}}_1 - \lambda_1) \) and \( \hat{\theta}_1 = (\theta^{\hat{\theta}}_1 - \theta_1) \). \( \lambda_1 \) and \( \theta_1 \) are the estimates of \( \lambda_0 \) and \( \theta_0 \), respectively. The time derivative of \( V_1 \) is

\[
\dot{V}_1(z_1, \lambda_1, \dot{\lambda}_1) = z_1(x_2 + c_{\theta_1} \xi_1(x_1) \omega_1 - \dot{y}_d + \nu_1)
\]

(24)

Let \( \gamma_1 > 0 \), we can get

\[
c_{\theta_1} \xi_1(x_1) \omega_1 = c_{\theta_1} \xi_1(x_1) \omega_1 + \gamma_1^2 \omega_1^T \omega_1
\]

\[
= -\gamma_1^2 (\omega_1^T \omega_1 - \frac{c_{\theta_1} \xi_1(x_1) \omega_1}{\gamma_1^2}) + \gamma_1^2 \omega_1^T \omega_1
\]

\[
\leq \frac{c_{\theta_1}^2}{\gamma_1^4} \xi_1^T \xi_1 \omega_1^2 + \frac{\gamma_1^2}{\gamma_1^2} \omega_1^T \omega_1
\]

\[
\leq \frac{c_{\theta_1}^2}{\gamma_1^4} \xi_1^T \xi_1 \omega_1^2 + \frac{\gamma_1^2}{\gamma_1^2} \omega_1^T \omega_1
\]

(25)

Noting (22), we get

\[
\nu_1 \| \leq \theta_1 \psi_1(x_1) \| \| \dot{z}_1 \| + \dot{\theta}_1 \psi_1(x_1) \| \| \dot{z}_1 \|
\]

(26)

Therefore, defining the error variable \( \dot{z}_2 = x_2 - \alpha_1 - \dot{y}_d \)

where \( \alpha_1 \) is the intermediate stabilizing function. We can get

\[
\alpha_1 = -k_1 z_1 - \frac{\lambda_1}{\gamma_1} \xi_1(x_1) \xi_1^T (x_1) \dot{z}_1
\]

(27)

where \( k_1 > 0 \) and \( \delta_1 > 0 \) are the design constants.

Substituting (25), (26), and (27) into (24), we get

\[
\dot{V}_1(z_1, \lambda_1, \dot{\lambda}_1) \leq -k_1 \dot{z}_1^2 + z_1 \dot{z}_1 + \dot{\theta}_1 \psi_1(x_1) \| \| \dot{z}_1 \| \| \theta_1 \| (x_1) \| \psi_1(x_1) \| \| \dot{z}_1 \| \leq \frac{\lambda_1}{\gamma_1} \xi_1^T \xi_1 \| \dot{z}_1 \| \| \theta_1 \| (x_1) \| \psi_1(x_1) \| \| \dot{z}_1 \|
\]

(28)

(29)

(30)

(31)

(32)

(33)

(34)

(35)
Choosing Lyapunov candidate function

\[ V_2 = V_1 + \frac{1}{2} \gamma_2^2 + \frac{1}{2} \hat{\gamma}_2^2 + \frac{1}{2} \hat{\gamma}_2^2. \]

A similar procedure with (25) and (26) is used and the time derivative of \( V_2 \) becomes

\[ \dot{V}_2 \leq -k_1 \Delta^2 + z_2 + \gamma_2^2 \hat{\Delta}^2 + \delta_1 \]

\[ + \frac{1}{2} \left( x_3 \frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_3} \right) \left( \frac{\gamma_2^2}{\gamma_2^2} \hat{\Delta}^2 - \delta_2 \right) \hat{\Delta}^2 + \frac{1}{2} \gamma_2^2 \hat{\Delta}^2 \]

where \( \delta_1 \) and \( \delta_2 \) are defined constants.

Now, choose the intermediate stabilizing function \( \alpha_2 \) and adaptive laws as

\[ \alpha_2 = -z_2 - k_2 \Delta^2 - \frac{1}{2} \left( z_2 - \Delta^2 - \delta_2 \right) \]

\[ = f(n-1)(z_{n-1}, x_n, \bar{x}_{d(n-1)}, w) + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \partial x_j + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_{d(j-1)}} y_d^{(j)} \]

Then

\[ \dot{z}_n = f(n)(z_{n-1}, x_n, \bar{x}_{d(n-1)}, w) + v + d_n - y_d^{(n)} \]

where \( f(n) = f_0(x, w) - f(n-1)(z_{n-1}, x_n, \bar{x}_{d(n-1)}, w) \).

We also use T-S fuzzy system to approximate the unknown function \( f(n)(z_{n-1}, x_n, \bar{x}_{d(n-1)}, w) \) and obtain

\[ f(n) = \xi_n \xi_n^{(1)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(2)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(3)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(4)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) \]

Taking the following Lyapunov function candidate

\[ V_n = V_{n-1} + \frac{1}{2} \gamma_n^2 + \frac{1}{2} \hat{\gamma}_n^2 + \frac{1}{2} \hat{\gamma}_n^2 \]

its time derivative is

\[ \dot{V}_n = \dot{V}_{n-1} + \gamma_n \dot{\xi}_n \xi_n^{(1)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(2)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(3)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(4)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) \]

where \( \gamma_n = \gamma_n \xi_n^{(1)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(2)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(3)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) + \xi_n \xi_n^{(4)}(z_{n-1}, x_n, \bar{x}_{d(n-1)}) \).
and adaptive laws in step n as
\[
\begin{align*}
\lambda_n &= \Gamma_{1n} \left[ \frac{1}{n} \sum_{i=1}^{n} \xi_i \sigma_n^2 - \sigma_1 (\lambda_n - \lambda_n^0) \right], \\
\theta_n &= \Gamma_{2n} \left[ \psi_n \| \zeta_n \| - \sigma_2 (\theta_n - \theta_n^0) \right].
\end{align*}
\]
(40)

According to the recursive control design procedure above, at the last step (i.e., \( i = n \)), picking the robust adaptive fuzzy control \( v = \alpha_n \) and the adaptive laws in (40), we arrive at
\[
\begin{align*}
\dot{V}_n &\leq -\sum_{i=1}^{n} k_i z_i^2 - \frac{1}{2} \sum_{i=1}^{n} (\sigma_i^2 \lambda_n^2 + \sigma_2^2 \theta_n^2) + \frac{1}{2} \sum_{i=1}^{n} \gamma_i^2 \omega_i^2 + \delta_n', \\
&\leq -\sum_{i=1}^{n} k_i z_i^2 - \frac{1}{2} \sum_{i=1}^{n} (\sigma_i \lambda_n^2 + \sigma_2 \theta_n^2) + \sum_{i=1}^{n} \gamma_i^2 \omega_i^2 + \delta_n',
\end{align*}
\]
(41)

where \( \delta_n' = \sum_{i=1}^{n} \xi_i + \delta_n + 1/2 \| \sigma_n \|_2^2 + 1/2 \| \theta_n - \theta_n^0 \|_2^2, \omega = [\omega_1, \omega_2, \ldots, \omega_n]^T \) and \( \gamma = (\gamma_1^2 + \gamma_2^2 + \cdots + \gamma_n^2)^{1/2} \).

We are now in a position to state our main result on semi-global robust adaptive fuzzy controller.

Theorem 2: Consider the system (13) with uncertain input gain term \( g_0(x, w) = 0 \), and suppose that the packaged uncertain functions \( f_i(x, w), i = 1, 2, \ldots, n \) can be approximated by T-S fuzzy systems in the sense that \( e_i \) is bounded. If we pick \( \gamma < 1 \) and \( k_i > 1, i = 1, 2, \ldots, n \) in (41), then the robust adaptive fuzzy control laws (40) and the intermediate stabilizing functions \( \alpha_i, i = 1, 2, \ldots, n \) and adaptive laws for \( \lambda_n \) and \( \theta_n \) make all the solutions of (41) uniformly ultimately bounded. Furthermore, given any \( \mu > 0 \), we can tune our controller parameters such that the output error \( z_1 = y(t) - y_d(t) \) satisfies \( \lim_{t \to \infty} | z_1(t) | \leq \mu \).

Proof: See Appendix I.

B. Robust Adaptive Fuzzy Tracking Sliding Control

In all of the above analyses, let the input uncertain \( g_0(x, w) \neq 0 \) be zero. Now we shall assume the input uncertain nonlinear system with the transformation in (17)
\[
\begin{align*}
\dot{x}_i &= x_{i+1} + f_i(x_i, w) + d_i, \\
\dot{x}_n &= F(x, w) + (1 + E(x, w)) v + d_n, \\
x &= x_1
\end{align*}
\]
(42)

where \( F(x, w) = f_0(x, w) - (1 + g_0^{-1}(x)) g_0(x, w) f_0(x) \).

Sliding mode techniques, yielding robust control, and adaptive control techniques are both popular when there is uncertainty in the plant. The combination of these methods has been studied in recent years [47], [48]. In general, at each step of the backstepping method, the new update tuning function takes the system to equilibrium position. At the final step, the system is stabilized by suitable selection of the control.

The adaptive sliding backstepping control of strict-feedback systems has been studied by [49], [50]. The controller is based upon sliding backstepping mode techniques so that the state trajectories approach a specified hyperplane. The sufficient condition (for the existence of the sliding mode) given by [51], is no longer needed.

To provide robustness, the adaptive backstepping algorithm can be modified to yield adaptive sliding output tracking controllers. The modification is carried out at the final step of the algorithm by incorporating the following sliding surface defined in terms of the error coordinates
\[
s = c_1 z_1 + c_2 z_2 + \cdots + c_{n-1} z_{n-1} + z_n
\]
(43)

where \( c_i > 0, i = 1, 2, \ldots, n - 1 \), are real numbers.

\[
\begin{align*}
\frac{\dot{z}_i}{s} &= z_{i+1} + \alpha_i (x_i, w) + d_n - \delta_n, \\
\frac{s}{s} &= F(x, w) + (1 + E(x, w)) v + \eta + \nu_n
\end{align*}
\]
(44)

where \( \eta = \sum_{i=1}^{n-1} c_i z_{i+1} - f_{(n)(n-1)} x_{n-1}, z_{n-1} \), \( \nu_n = d_n - y_d - \sum_{i=1}^{n-1} \partial_{\eta n-1} \partial_{y_d} d_j + \partial_{\eta n-1} \partial_{y_d} y_d(j) \).

For the system (44), when the controller is designed based on the feedback linearization technique, the most commonly used control structure is \( v = -1/2ks + F(x) + \eta / (1 + E(x)) \) if there are no uncertain parameters in the system. When there are uncertainties in the system, the nonlinearities \( F(x, w), (1 + E(x, w)) \) are unknown and many adaptive control schemes have been developed, in which the unknown function \( (1 + E(x, w)) \) is usually approximated by a function approximator \((1 + E(x))\). Consequently, the estimate \((1 + E(x))\) must be away from zero to avoid a possible singularity problem. We will solve this problem based on a novel Lyapunov function [52] as follow
\[
V_s(s) = \int_0^s \left( \frac{\sigma}{1 + E(w, \psi, \sigma + \eta)} \right) d\sigma
\]
(45)

where \( x_n = s + c_1 \eta_{(n-1)}(t) - \sum_{i=1}^{n-1} c_iz_i = s + \eta_1, \psi = [x_1, x_2, \ldots, x_{n-1}]^T \).

By mean value theorem, \( V_s \) can be rewritten as
\[
V_s(s) = \frac{\lambda_s}{(1 + E(w, \psi, \lambda_s(s + \eta))}
\]

where \( \lambda_s \in (0, 1) \). By Assumption 1, we can get \( (1 + E(x, w)) \geq (1 + b_{\min}) > 0 \) such that \( 1/(1 + E(x, w)) > 0 \). Then it is shown that \( V_s \) is positive definite with respect to \( s \).

Additionally, the Lyapunov candidate function is modified as follows
\[
V_n = V_{n-1} + V_s + \frac{1}{2} \left[ 1 - n \lambda_n^2 + \frac{1}{2} \gamma_0 \right] \theta_n^2
\]

For Lyapunov function candidate \( V_{n-1} \), its time derivative is
\[
\dot{V}_n = \dot{V}_{n-1} + \dot{V}_s - n \lambda_n \dot{\lambda}_n - \frac{1}{2} \gamma_0 \dot{\theta}_n \theta_n
\]
(46)

where
\[
\dot{V}_s = \frac{\partial V_s}{\partial s} s + \frac{\partial V_s}{\partial \psi} \psi + \frac{\partial V_s}{\partial \eta} \eta
\]

\[
= \frac{s}{1 + E(x, w)} s + \int_0^s \frac{\partial (1 + E(x, w))}{\partial \psi} \psi d\sigma
\]

\[
+ \eta \int_0^s \frac{\partial (1 + E(x, w))}{\partial \eta} \psi d\sigma
\]
Since \( \eta = -\dot{r}_1 \), \( (1+E(w,\psi,\sigma+\eta))^{-1}/\partial \eta_1 \) is given in (49), can be approximated by T-S fuzzy systems. If we get sliding surface (43), pick \( \zeta = \left[ \frac{\tau_{n-1}}{s_i} \right]^T \), see equation at the bottom of the page. From (43) we obtain
\[
\zeta_n = s - \sum_{i=1}^{n-1} c_i \zeta_i. 
\]

As the function \( F(x,w)/(1+E(x,w) + h(\zeta)) \) in (47) is an unknown continuous function, based on Lemma 1, the following Takagi–Sugeno fuzzy system can be used to approximate it
\[
F(x,w) = \xi_n(\zeta) A_{n} s + e_n = \xi_n(\zeta) A_{n} \left[ \frac{\tau_{n-1}}{s} \right] + \Delta_n = c_{0n} \xi_n(\zeta) w_n + \Delta_n. 
\]

Substituting (43) and (44) into (42), we have
\[
\dot{V}_n \leq - \sum_{i=1}^{n-1} k_{i} \dot{z}_i^2 - \frac{1}{2} \sum_{i=1}^{n-1} (\sigma_1 \chi_i^2 + \sigma_2 \theta_i^2) + \sum_{i=1}^{n} \gamma_i^2 \omega_i^T \omega_i \\
+ s \left[ v + \frac{F(x,w)}{1+E(x,w)} + h(\zeta) + \frac{\nu_n}{1+E(x,w)} \right] \\
- \sum_{i=1}^{n-1} \delta'_i - \zeta_n - \zeta_n^{1/2} \Delta_n \\
\]
where \( \nu_n = d_n + v'/(1+E(x,w)) \).

A similar procedure with (25) and (26) is used and the time derivative of \( V_n \) becomes
\[
V_n' \leq - \sum_{i=1}^{n-1} k_{i} \dot{z}_i^2 - \frac{1}{2} \sum_{i=1}^{n-1} (\sigma_1 \chi_i^2 + \sigma_2 \theta_i^2) + \sum_{i=1}^{n} \gamma_i^2 \omega_i^T \omega_i \\
+ \sum_{i=1}^{n} \delta'_i - \zeta_n - \zeta_n - \zeta_n^{1/2} \Delta_n \\
+ \left[ v + \frac{F(x,w)}{1+E(x,w)} + h(\zeta) + \frac{\nu_n}{1+E(x,w)} \right] \\
- \zeta_n - \zeta_n^{1/2} \Delta_n \\
\]
where \( \nu_n = \left[ k_{1} 0 \cdots 0 \\ 0 k_{2} \cdots 0 \\ \vdots \vdots \vdots \\ 0 \cdots k_{n-1} 0 \right] \) and
\[
\Delta_n = \left[ c_{1} c_{2} \cdots c_{n-1} \right] \left[ k_{1} 0 \cdots 0 \\ 0 k_{2} \cdots 0 \\ \vdots \vdots \vdots \\ 0 \cdots k_{n-1} \right] \left[ 0 \cdots 0 \right]. 
\]

Theorem 3: Consider the system (13) with uncertain input gain term \( g_i(x,w) \neq 0 \), and suppose that Assumption 1 is satisfied and the packaged uncertain functions \( f_i(x_{i-1},x_i,x_{d_i},w). \)
\( i = 1, 2, \ldots, n \), where \( f_i \), is given in (49), can be approximated by T-S fuzzy systems. If we get sliding surface (43), pick \( \gamma < 1 \) and \( k_i \geq 1, i = 1, 2, \ldots, n \), then the control scheme
\[
h(\zeta) = \frac{1}{s} \int_0^s \left[ \frac{\sigma_1 \chi_i^2 + \sigma_2 \theta_i^2 + \sum_{i=1}^{\infty} \gamma_i^2 \omega_i^T \omega_i}{1+E(x,w)} + \frac{\eta}{1+E(x,w)} \right] \, \mathrm{d}t. 
\]
\[ u = g_0^{-1}(x(-f_0(x) + v)), \]
where \( v \) is given in (52) with adaptive laws in (53) and the intermediate stabilizing functions \( \alpha_i, \]
\( i = 1, 2, \ldots, n - 1 \) and adaptive laws for \( \lambda_i \) and \( \hat{\theta}_i \), is a robust adaptive fuzzy sliding tracking controller (RAFTC) which can make all the solutions \((z(t), \lambda, \hat{\theta})\) of the derived closed loop system uniformly ultimately bounded. Furthermore, given any \( \mu > 0 \), we can tune our controller parameters such that the output error \( z_1 = y(t) - y_d(t) \) satisfies \( \lim_{t \to \infty} |z_1(t)| \leq \mu \).

**Proof:** This proof is similar to that of Theorem 2.

V. APPLICATION EXAMPLES

Now we will reveal the control performance of the proposed RAFTC and RAFSTC via application examples. In this section, we present four examples. The first one is a pendulum system plus driven motor. We design a RAFTC for this system. The second application is a one-link robot system with the inclusion of motor dynamics. Owing to unknown function \( g_k \) in the one-link robot system, we use Theorem 3 to design RAFSTC for it. The third application is that a RAFSTC is applied for ship roll stabilization with actuator. In the last one, we apply RAFSTC to design a controller for a single-link manipulator tracking with flexible joint.

These applications demonstrate that RAFTC and RAFSTC proposed in this paper have great potential in many diverse applications.

A. Pendulum System With Motor

Consider the following dynamic system:

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3 - a \sin(x_1) - q \cos(x_1) + d_1(t), \\
\dot{x}_3 &= -x_3 + u + d_2(t)
\end{align*}
\]  
(55)

where \( x_1 \) and \( x_2 \) denote the angular position and rate of the pendulum, and \( x_3 \) denotes the the motor shaft angle. \( u \) is the control input used to represent the motor torque. \( a \) and \( q \) are the system’s unknown parameters, and \( d_1(t) \) and \( d_2(t) \) both are the uncertain terms of the system’s external disturbance.

The pendulum plus driven motor is a simple system. This model was employed at the first time by Gutman [53] when he discussed the design of robust control of uncertain nonlinear system, but he didn’t take the influence of driven motor (that is, the third line in the above equation). Kwan [54] also had studied this system, without consideration of the influence of external uncertain disturbance term \( d_2(t) \).

Define five fuzzy sets for each variable \( x_1, z_1, z_2, y_d \) and so on with labels \( A_{11}^i(\text{NL}), A_{12}^i(\text{NM}), A_{13}^i(\text{ZE}), A_{14}^i(\text{PM}), A_{15}^i(\text{PL}) \) which are characterized by the following membership functions

\[
\begin{align*}
\mu_{A_{11}^i}(x) &= \exp[-(x + 1)^2] \\
\mu_{A_{12}^i}(x) &= \exp[-(x + 0.5)^2] \\
\mu_{A_{13}^i}(x) &= \exp[-x^2] \\
\mu_{A_{14}^i}(x) &= \exp[-(x - 0.5)^2] \\
\mu_{A_{15}^i}(x) &= \exp[-(x - 1)^2].
\end{align*}
\]  
(56)

We can use Theorem 2 to design the robust adaptive fuzzy tracking controller for this system. The stabilizing functions \( \alpha_1 \) and \( \alpha_2 \) are

\[
\begin{align*}
\alpha_1 &= -4z_1, \\
\alpha_2 &= -z_1 - 20z_2 - \lambda_2 z_2^2 - \hat{\theta}_2 \psi_2 \tanh\left(\frac{\hat{\theta}_2 \psi_2 z_2}{100}\right)
\end{align*}
\]

where \( z_1 = x_1 - y_d \) and \( z_2 = x_2 - \alpha_1 - y_d \). Adaptive laws for \( \lambda_2 \) and \( \hat{\theta}_2 \) are

\[
\begin{align*}
\dot{\lambda}_2 &= 75 \left[\lambda_2 z_2^2 - 0.03(\lambda_2 - 0.1)\right] \\
\dot{\hat{\theta}}_2 &= 6 \left[\psi_2 \| z_2 \| - 0.3(\hat{\theta}_2 - 0.1)\right]
\end{align*}
\]

and we obtain the controller law as

\[
\begin{align*}
\dot{z}_2 &= -2z_2 - 8z_3 - \lambda_3 z_3^2 z_3 - \hat{\theta}_3 \psi_3 \tanh\left(\frac{\hat{\theta}_3 \psi_3 z_3}{100}\right)
\end{align*}
\]  
(57)

where \( z_3 = x_3 - \alpha_2 - y_d \).

The stabilizing functions \( \alpha_3 \) and \( \alpha_4 \) are

\[
\begin{align*}
\alpha_3 &= -4z_3, \\
\alpha_4 &= -z_3 - 20z_4 - \lambda_4 z_4^2 - \hat{\theta}_4 \psi_4 \tanh\left(\frac{\hat{\theta}_4 \psi_4 z_4}{100}\right)
\end{align*}
\]

where \( z_4 = x_4 - y_d \).

Then adaptive laws for \( \lambda_3 \) and \( \hat{\theta}_3 \) are

\[
\begin{align*}
\dot{\lambda}_3 &= 0.15 \left[\lambda_3 z_3^2 - 0.005(\lambda_3 - 0.1)\right] \\
\dot{\hat{\theta}}_3 &= 0.02 \left[\psi_3 \| z_3 \| - 0.06(\hat{\theta}_3 - 0.01)\right].
\end{align*}
\]

In simulation, we employ the following system parameters:

\[
\begin{align*}
\alpha = 11, \quad q = 2, \quad d_1(t) &= \sin(t), \quad d_2 = 5\sin(t) \quad \text{The initial conditions for } x_1, x_2, x_3 \text{ are } 0.2, 2\pi \text{ and } 0.
\end{align*}
\]

Figs. 1–3 show the simulation results with the reference signal \( y_d = \sin(\pi t) \). From Figs. 1 and 2, we can see that a large tracking error exists during the first 3 s. This is due to the lack of knowledge about the plant.
nonlinearities. Through the adaptive laws learning phase, better tracking performance is obtained after 3 s. Figs. 4 and 5 show the adaption of parameters in RAFTC algorithm.

B. One-Link Robot Tracking

Consider a one-link manipulator with the inclusion of motor dynamics. The robot model is

\[
\dot{q} + \beta \dot{q} + N \sin(q) = \tau + \tau_d
\]

where \( q, \dot{q}, \ddot{q} \) denote the link position, velocity, and acceleration, respectively. \( \tau \) and \( \tau_d \) are the motor shaft angle and velocity. \( \tau_d \) represents the torque disturbance. \( \dot{u} \) is the control input used to represent the motor torque. Above equation can be expressed in the form (45) by noting that

\[
x_1 = q, \, x_2 = \dot{q}, \, x_3 = \frac{\tau}{D}, \, f_1(x_1, w) = 0,
\]

\[
f_2(x_1, x_2, w) = \frac{-N \sin(x_1) - Bx_2}{D}, \quad F(x, w) = \frac{-K_m x_2 - HDx_3}{MD}, \quad 1 + E(x, w) = \frac{1}{MD}
\]

\[
d_1 = 0, \quad d_2 = \frac{\tau_d}{D}, \quad d_3 = 0.
\]

The parameter values with appropriate units are given by \( D = 1, M = 0.05, B = 1, K_m = 10, H = 0.5, N = 10 \). In order to exhibit the robustness of the proposed control schemes, we chose the torque disturbance \( \tau_d \) as the band-limited white noise with noise power 1 and sample time 1 in the simulation.

We can define some fuzzy sets for each variable as (56) and use Theorem 3 to design the robust adaptive sliding fuzzy tracking controller for this system. The first stabilizing functions \( \alpha_1 \) and \( \alpha_2 \) are

\[
\alpha_1 = -36z_1
\]

\[
\alpha_2 = -z_1 - 35z_2 - \lambda_2 \xi_2 \xi_3 e_2 - \hat{\theta}_2 \psi_2 \tanh \left( \frac{\hat{\theta}_2 \psi_2 z_2}{200} \right)
\]

with \( z_1 = x_1 - y_d \) and \( z_2 = x_2 - \alpha_1 - \hat{\theta}_2 \psi_2 \). And adaptive laws as

\[
\begin{cases}
\dot{\lambda}_2 = 0.01 \left[ \xi_3 \xi_2 \xi_3 e_3 - 0.000001 (\lambda_3 - 0.1) \right] \\
\dot{\lambda}_3 = 0.3 \left[ \psi_3 \| s \| - (\hat{\theta}_3 - 0.1) \right]
\end{cases}
\]

where \( e_3 = x_3 - \alpha_3 = \dot{y}_d \) and the sliding surface is \( s = z_3 + 2z_1 + 5.5z_2 \). Then adaptive laws are

\[
\begin{cases}
\dot{\lambda}_2 = 0.01 \left[ \xi_3 \xi_2 \xi_3 e_3 - 0.000001 (\lambda_3 - 0.1) \right] \\
\dot{\lambda}_3 = 0.3 \left[ \psi_3 \| s \| - (\hat{\theta}_3 - 0.1) \right]
\end{cases}
\]

We apply controller (59) to one-link robot system for tracking a desired trajectory \( y_d = \sin(2\pi t) \). The simulation results are shown in Fig. 6–8.

C. Ship Roll Stabilization With Actuator

In this subsection, we will apply RAFSTC to a typical ship control problem, that is, ship fin roll stabilization. The ship roll
Fig. 6. Simulation results for one-link robot system: System output $y$ and reference signal $y_d$ (solid line: $y$ and dashed line: $y_d$).

Fig. 7. Simulation results for one-link robot system: Tracking error $z_1$.

Fig. 8. Simulation results one-link robot system: Control $u$.

where $\rho$ is the density of water, $V$ denotes ship speed, $A_f$ is the area of the fin, $C_L^o$ is the slope of lift coefficient, $I_f$ is the arm of force supplied by fin, $\alpha_f$ is the fin angle.

In roll stabilization system, the actual fin angle $\alpha_f$ is driven by a mechanical equipment which is called actuator. The actuator consists of electrical-hydraulic system. Generally speaking, the actual fin angle $\alpha_f$ has a time-delay compared with order fin angle. Here, we use a 1st order inertia system to describe the actuator’s dynamical property as follows

$$T_E \dot{\alpha}_f + \alpha_f = K_E \alpha_c$$  \hspace{1cm} (61)

where $\alpha_c$ is order fin angle (i.e., controller’s output), $\alpha_f$ is actual fin angle, $T_E$ denotes the time constant of actuator, and $K_E$ is input gain.

To illustrate the applicability of the robust adaptive fuzzy sliding tracking control scheme proposed in this paper, we conduct a simulation on the ship roll stabilization. Let the state variables be $x_1 = \varphi, x_2 = \dot{\varphi}, x_3 = (\rho V^2 A_f C_L^o I_f / I_{xx} + J_{xx}) \alpha_f$ and control variable be $u = \alpha_c$. Then the mathematical model for ship roll by fin control with actuator can be written in a general model of typical strict-feedback nonlinear system as follows

$$\begin{align*}
    \dot{x}_1 &= \dot{\varphi} \\
    \dot{x}_2 &= f_2(x_1, x_2) + x_3 + d_2 \\
    \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)u + d_3
\end{align*}$$ \hspace{1cm} (62)

where $f_2 = -D_h x_1 + N x_2 - W x_2 | x_2 | - \rho V^2 A_f C_L^o I_f / I_{xx} + J_{xx}; f_3 = -1 / T_{E_x} x_3$.

Define five fuzzy sets for each state error variable $z_1, z_2, z_3$ with labels $A^1_h(NL), A^2_h(NM), A^3_h(ZE), A^4_h(PM), A^5_h(PL)$ which are characterized by the following membership functions

$$\begin{align*}
    \mu_{A^1_h} &= \exp \left[-\frac{z - \frac{\pi}{6} - \varphi}{\frac{\pi}{21}}\right]^2 \\
    \mu_{A^2_h} &= \exp \left[-\frac{z + \frac{\pi}{12}}{\frac{\pi}{21}}\right]^2 \\
    \mu_{A^3_h} &= \exp \left[-\frac{z - \frac{\pi}{12}}{\frac{\pi}{21}}\right]^2 \\
    \mu_{A^4_h} &= \exp \left[-\frac{z - \frac{\pi}{6}}{\frac{\pi}{21}}\right]^2 \\
    \mu_{A^5_h} &= \exp \left[-\frac{z + \frac{\pi}{6}}{\frac{\pi}{21}}\right]^2. \hspace{1cm} (63)
\end{align*}$$

According to Theorem 3, the intermediate stabilizing functions $\alpha_1$ and $\alpha_2$ are

$$\begin{align*}
    \alpha_1 &= -1.0 z_1 \\
    \alpha_2 &= -z_1 - 10.0 z_2 - \lambda_2 \dot{\psi}_2^T z_2 - \dot{\theta}_2 \psi_2 \tanh \left(\frac{\dot{\theta}_2 \psi_2}{0.1}\right)
\end{align*}$$
and adaptive laws as
\[
\dot{\lambda}_2 = 10 \left[ \xi_2 \xi_2^T \dot{z}_2 - 0.5(\lambda_2 - 0.1) \right]
\]
\[
\dot{\theta}_2 = 2 \left[ \psi_2 \| z_2 \| - 0.5(\dot{\theta}_2 - 0.1) \right]
\]
where \( \gamma = 0.5 \), \( z_1 = x_1 \), \( z_2 = x_2 - \alpha_1 \) and \( z_3 = x_3 - \alpha_2 \).

The control law is
\[
u = -z_2 - 0.8s - \lambda_3 \xi_3 \xi_3^T s - \dot{\theta}_3 \psi_3 \tanh \left( \frac{\dot{\theta}_3 \psi_3}{0.1} \right)
\]
where the sliding mode surface \( s = z_3 + 3z_1 + 2z_2 \) and adaptive laws as
\[
\dot{\lambda}_3 = 25 \left[ \xi_3 \xi_3^T \dot{s} - 0.5(\lambda_3 - 0.1) \right]
\]
\[
\dot{\theta}_3 = 10 \left[ \psi_3 \| s \| - 0.5(\dot{\theta}_3 - 0.1) \right].
\]

To verify the feasibility of the proposed adaptive fuzzy control scheme for ship roll stabilization, we use a container ship with the length 175 m and displacement 25,000 tons as an example to conduct a simulation research. In the simulation, the external disturbance is assumed as a sinusoidal wave with wave height 7 m and wave direction 30°.

Fig. 9 illustrates the time response of the ship roll angle without fin control. Fig. 10 shows the time response of the ship roll angle with RAFSTC in (64). Fig. 11 gives the time response of fin control angle \( \alpha_c \).

**D. Single-Link Manipulator Tracking With Flexible Joint**

The dynamics for a single-link manipulator with flexible joint [55] is described by
\[
I \ddot{q}_1 + MgL \sin q_1 + K(q_1 - q_2) = 0
\]
\[
J \ddot{q}_2 - K(q_1 - q_2) = u
\]
where \( q_1, \dot{q}_1, \ddot{q}_1 \) denote the link angular position, velocity, and acceleration, respectively. The parameters described by \( I \) and \( J \) in the system are moments of inertia. \( M \) is the mass, \( L \) is the length of link, \( K \) is the parameter which characterize the joint flexibility, \( q_2, \dot{q}_2, \ddot{q}_2 \) are the motor shaft angle, velocity, acceleration, respectively, \( u \) is the control input used to represent the motor torque.

If we define \( x_1 = q_1, x_2 = \dot{q}_1, x_3 = (K/I)q_2, x_4 = (K/I)\dot{q}_2 \), then (65) is indeed in the form of (42). The control objective is to develop a link position-tracking controller for the single-link manipulator with flexible joint given by (65), based on inexact knowledge of manipulator dynamics. To accomplish this purpose, we first define the link position tracking error \( z_1 \) as
\[
z_1 = x_1 - q_d
\]
where \( q_d \) is the link position reference trajectory. It is assumed that reference model for \( q_d \) is chosen so as to represent somewhat realistic performance requirement as
\[
\dot{q}_d + 1.4q_d + q_d = r(t)
\]
where \( r(t) \) is the desired signal.

![Fig. 9. Simulation results for time response of roll angle without fin control.](image1)

![Fig. 10. Simulation results for time response of roll angle with fin control.](image2)

![Fig. 11. Simulation results for time response of fin control angle.](image3)

Same fuzzy sets for each variable as (56) and Theorem 3 are used to design the robust adaptive fuzzy tracking controller for this system. The stabilizing functions \( \alpha_1, \alpha_2, \) and \( \alpha_3 \) are
\[
\alpha_1 = -7z_1
\]
\[
\alpha_2 = z_2 - 10z_2 - 0.69\lambda_2 \xi_2 \xi_2^T z_2 - \dot{\theta}_2 \psi_2 \tanh \left( \frac{\dot{\theta}_2 \psi_2}{20} \right)
\]
\[
\alpha_3 = z_2 - 15z_3 - \lambda_3 \xi_3 \xi_3^T z_3 - \dot{\theta}_3 \psi_3 \tanh \left( \frac{\dot{\theta}_3 \psi_3}{2} \right)
\]
where \( z_2 = x_2 - \alpha_1 - \dot{q}_d \) and \( z_3 = x_3 - \alpha_2 \). Adaptive laws are
\[
\begin{align*}
\dot{\lambda}_2 &= 7\xi_2 \xi_2^T - 0.02(\lambda_2 - 0.1) \\
\dot{\theta}_2 &= 1[\psi_2 \| z_2 \| - 0.08(\dot{\theta}_2 - 0.01)] \\
\dot{\lambda}_3 &= 0.69\xi_3 \xi_3^T - 0.001(\lambda_3 - 0.1) \\
\dot{\theta}_3 &= 5[\psi_3 \| z_3 \| - 0.05(\dot{\theta}_3 - 0.01)]
\end{align*}
\]
and reference signal, which changes its value in square form. The controller is trained.

We get the controller law as

\[ u = z_3 - 120s - \lambda_4 \xi_4^T s - \theta_4 \psi_4 \tanh \left( \frac{\theta_4 \psi_4 s}{100} \right) \]

where \( z_4 = x_4 - \alpha_3 \) and \( s = z_4 + 25z_1 + 3z_2 + 5z_3 \). Then adaptive laws are

\[
\begin{align*}
\dot{\lambda}_4 &= 25\xi_4 \xi_4^T s^2 - 0.001(\lambda_4 - 0.1) \\
\dot{\theta}_4 &= 5[\psi_4 \lvert s \rvert - 0.07(\theta_4 - 0.01)]
\end{align*}
\]

In the simulation, the parameter values are \( M = 2.3 \) kg, \( L = 1 \) m, \( g = 9.8 \) m/s² and \( I = MI^2 \). The flexible joint parameters are \( J = 0.5 \) and \( K = 15 \). The controller is trained by applying a signal \( r(t) \), which changes its value in square form with the interval \([-1, 1]\) in the frequency 0.3 rad/sec. We give the simulation results in Figs. 12-14.

The simulation results shown in Figs. 12 and 13 indicate that the tracking error is lower than 2.5 percentages. These results also indicate that the developed controller can be applied to the uncertain single-link manipulator system with flexible joint.

VI. CONCLUSION

In this paper, the tracking control problem has been considered for a general class of strict-feedback uncertain nonlinear systems. We have discussed that the systems possess a wide class of uncertainties referred to as unstructured uncertainties, which are not linearly parameterized and have no prior knowledge of the bounding functions, and used Takagi–Sugeno type fuzzy logic systems to approximate unstructured uncertain functions. Combining backstepping technique with small-gain approach, we have proposed a robust adaptive fuzzy tracking control (RAFTC) algorithm for the system without input gain uncertainty and a robust adaptive fuzzy sliding tracking control (RAFSTC) algorithm for the system with input gain uncertainty. And both algorithms can guarantee that the closed-loop system is semi-globally uniformly ultimately bounded. The main feature of the algorithms proposed is the adaptive mechanism with minimal learning parameterizations, that is, no matter how many states in the system are investigated and how many rules in the fuzzy system are used, only \( 2n \) parameters are needed to be adapted online. Then the computation load of the algorithm can be reduced, and it is a convenience to realize this algorithm for engineering. Finally, four application examples have been presented to illustrate the tracking and stabilization performance of the closed-loop systems by use of the proposed RAFTC and RAFSTC algorithms.

APPENDIX

PROOF OF THEOREM 2

In order to use Theorem 1 (Small-gain theorem), it is necessary to construct a system in composite feedback form with \( \Sigma_{\omega} \)-subsystem and \( \Sigma_{\omega} \)-subsystem. We begin with the \( \Sigma_{\omega} \)-subsystem. According to the error variables \( z_i = x_i - \alpha_{i-1} - y_i \psi_i(\omega_i) \), \( i = 1, 2, \ldots, n \). \( \alpha_0 = 0 \) defined in Section IV, we substitute \( z_i \) into (13) and use T-S fuzzy systems to approximate the packaged uncertain functions \( f_i(z_{i-1}, x_i, \xi_i(k)), w) \), \( i = 1, 2, \ldots, n \), then the closed loop system can be given as follows

\[
\Sigma_{\omega} : \begin{cases}
\dot{z}_i = \alpha_i + z_{i+1} + c_{pi} \xi_i \omega_i + \nu_i, & 1 \leq i < n - 1 \\
\dot{z}_n = v + c_{n} \xi_n \omega_n + \nu_n \\
\dot{z} = H(z) = z
\end{cases}
\]

where \( \omega = [\omega_1, \omega_2, \ldots, \omega_n]^T \) is considered as the virtual input and \( z \) as the output.

For subsystem \( \Sigma_{\omega} \), if picking \( k_i > 1, i = 1, 2, \ldots, n \) from (41), we obtain

\[
\dot{V}_n \leq -s^2 + \xi_i \omega_i + k_i \omega_i.
\]

By Definition 2, we propose the robust adaptive fuzzy tracking controller such that the requirement of ISPFS for system \( \Sigma_{\omega} \) can be satisfied with the functions \( \alpha_3(s) = s^2 \) and
\[ \alpha_4 = \gamma^2 \sigma^2 \text{ of class } K_{\infty}. \] According to (4), we can get a gain function \( \gamma_c(s) \) of \( \Sigma_{w,0} \)-subsystem

\[ \gamma_c(s) = \alpha_1^{-1} \circ \alpha_2^{-1} \circ \alpha_3^{-1} \circ \alpha_4 \forall s > 0 \]

where \( \alpha_1(z) \leq V_n(z) \leq \alpha_2(z). \)

For \( \Sigma_{w,0} \)-subsystem, it is

\[
\Sigma_{w,0} : \begin{cases}
\omega_1 = A_{w}^{m} z_1 \\
\omega_2 = A_{w}^{m} [z_1, z_2]^T - A_{w}^{m} z_2 \\
\vdots \\
\omega_n = A_{w}^{m} [z_1, z_2, \ldots, z_n]^T = A_{w}^{m} z_n.
\end{cases}
\]

(A.2)

We can rewrite the above equations as

\[
\omega = \begin{bmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_n
\end{bmatrix} = K(z)
\]

\[
= \begin{bmatrix}
A_{1w}^{m} & 0 & \cdots & 0 \\
A_{2w}^{m} & A_{1w}^{m} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
A_{nw}^{m} & A_{2w}^{m} & \cdots & A_{1w}^{m}
\end{bmatrix}\begin{bmatrix}
z_1 \\
z_2 \\
\vdots \\
z_n
\end{bmatrix} = A z
\]

and obtain

\[
\| \omega \| \leq A \| z \| \Rightarrow \gamma' \| z \|.
\]

(A.3)

Then the gain function \( \gamma_w \) for system \( \Sigma_{w,0} \) is \( \gamma_w(s) = \gamma' s \).

In order to check the requirement \( \gamma'(\gamma_w(s)) < s \) in small gain theorem 1, we select (A.1) as (5), and (A.2) as (6), and obtain \( \gamma'_1 < 1 \). Due to \( \gamma' = \| A \| \leq 1 \), the condition of small gain theorem 1 can be satisfied by choosing \( \gamma < 1 \), such that it can be proven that the composite closed-loop system is ISP. Therefore, direct use of Definition 1 yields that the composite closed-loop system has bounded solutions over \([0, \infty)\). More precisely, there exists a class \( K_L \)-function \( \beta \) and a positive constant \( d_1 \) such that

\[
\| z(t), \lambda(t), \hat{\theta}(t) \| = \beta(\| z(0), \lambda(0), \hat{\theta}(0) \|, t) + d_1
\]

where \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T \) and \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_n]^T \).

This, in turn, implies that the tracking error \( z(t) \) is bounded over \([0, \infty)\). According to Proposition 1, there exists an ISP-Lyapunov function for the composite closed-loop system. By substituting (A.2) into (41), the ISP-Lyapunov function is satisfied as follows

\[
\dot{V}_n \leq -z^T Q z - \frac{1}{2} \lambda^T Q \lambda - \frac{1}{2} \hat{\theta}^T Q \hat{\theta} + \gamma' \| z \|^2 + \delta_n
\]

\[
\leq -z^T Q z - \frac{1}{2} \lambda^T Q \lambda - \frac{1}{2} \hat{\theta}^T Q \hat{\theta} + \| z \|^2 + \delta_n
\]

\[
\leq -c_1 V_n + \delta_n
\]

(A.4)

where \( Q = \text{diag}[Q_1, Q_2, \ldots, Q_n], Q_1 = \text{diag}[\sigma_{11}, \sigma_{12}, \ldots, \sigma_{1n}], Q_2 = \text{diag}[\sigma_{21}, \sigma_{22}, \ldots, \sigma_{2n}], \)

\[
\sigma_{1n}^T, Q_1 = \text{diag}[\sigma_{11}, \sigma_{12}, \ldots, \sigma_{1n}], Q_2 = \text{diag}[\sigma_{21}, \sigma_{22}, \ldots, \sigma_{2n}], c_1 = \min(2\lambda_{\text{min}}(Q_1), 1), \lambda_{\text{min}}(Q_2)_{\text{max}}(Q_1)), \gamma_1 = [\Gamma_{11}, \Gamma_{12}, \Gamma_{13}, \ldots, \Gamma_{1n}]^T \text{ and } \Gamma_2 = [\Gamma_{21}, \Gamma_{22}, \Gamma_{23}, \ldots, \Gamma_{2n}]^T. \]

From (A.4), we obtain

\[
V_n(t) \leq \frac{\delta_n}{c_1} + \left( V_n(t_0) - \frac{\delta_n}{c_1} \right) e^{-c_1(t-t_0)}.
\]

It results that the solutions of composite closed-loop system are uniformly ultimately bounded and implies that, for any \( \mu_1 > (\delta_n/c_1)^{1/2} \), there exists a constant \( T > 0 \) such that

\[
\| z(t) \| \leq \mu_1 \text{ for all } t \geq t_0 + T.
\]

The last statement holds readily since \((\delta_n/c_1)^{1/2}\) can be made arbitrarily small if the design parameters \( \lambda^0, \theta^0, \sigma_1, \sigma_2 \) are chosen appropriately. Finally, we have proved Theorem 2.

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